Mobile Robot Trajectory Tracking Based on Fast Terminal Sliding Mode Control

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ABSTRACT—This paper presents a trajectory tracking control method for nonholonomic mobile robot which is based only on kinematic model. A mobile robot trajectory tracking control method, based on terminal sliding mode control, with asymptotic stability is presented. Based on kinematic model of mobile robot, a terminal sliding mode control law for the angular velocity is proposed to reduce the tracking errors of the heading angle by using the finite time control method. Fast Terminal sliding mode control (FTSM) can conduct the system states to converge to zero in a finite time, ensures asymptotical convergence of states by using the Lyapunov stability method and proves that the sliding mode tracking controller is stable. There is no switch function in fast terminal sliding mode; therefore, the chattering phenomenon is avoided. The simulations results show that the control laws of this approach are tending towards to the desired velocity and show a stability in a very short time, thus the tracking errors of the planar coordinate and the errors of the heading angle are almost zero.

Keywords—mobile robot; terminal sliding mode; trajectory tracking; Lyapunov stability; finite time.

I. INTRODUCTION

Much research has been done on the tracking trajectory control of mobile robots [1], [2], it is known that stabilization of nonholonomic wheeled mobile robots with restricted mobility to an equilibrium state [3], identifies nonholonomic systems as a class of systems that cannot be stabilized via smooth state feedback. The kinematic equations of mobile robot for the tracking control can be classified into cartesian coordinates [4].

Classical sliding mode control has been shown to be robust and effective control approach for stabilization of nonlinear systems [5], [6]. The traditional method is based on the proposition of an exponentially stable sliding surface as a function of the system states and time and using the Lyapunov theory to ensure that all closed-loop system trajectories reach this surface in a finite time. The main disadvantage of the sliding mode control is in the chattering effect in real implementations when the system trajectories exhibit high-frequency chattering around the sliding surface due to the delay in switching the control values. The sliding mode controllers for higher-order sliding surfaces typically allow reducing chattering effect while providing finite-time converges to the sliding surface [7], [8].

An approach known as terminal sliding mode control (TSMC), firstly introduced in [9], presents the design of non-smooth sliding surfaces to ensure that the closed-loop system trajectories not only reach the sliding surface in finite time but also, during the sliding phase, converge to the origin in finite time. The main advantage of this method, compared with the traditional method, lies in the convergence time which is very short. A fast terminal sliding mode control law is proposed for solving the trajectory tracking problems of nonholonomic mobile robots and this control law can make converge the actual robot coordinate to the desired one in a finite time with asymptotic stability.

II. KINEMATICS MODELING OF MOBILE ROBOT

In this research a unicycle type wheeled mobile robot is used for the proposed algorithm, it has two driving wheels which are controlled by electric motors. The mobile robot posture can be presented by the position which is the middle point of the two driving wheels and the heading direction. Fig. 1 shows the position of the robot expressed in the X-Y coordinates. One can suppose that the posture vector of mobile robot is presented as \( p = (x, y, \theta)^T \), here \( (x, y) \) denotes the position of mobile robot and \( \theta \) is defined as the angle between the X-coordinate and the heading direction.

Fig. 1. Posture error description
The mobile robot motion is controlled by the vector \( q = (v, \omega)^T \); here \( v \) is the linear velocity of the robot and \( \omega \) is the angular velocity. According to the kinematics, the relationship between the posture vector \( p \) expressed in the X-Y coordinate and the velocity vector is derived as:

\[
\dot{p} = \begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
cos\theta & 0 & 0 \\
\sin\theta & 0 & 1 \\
0 & 0 & 1
\end{bmatrix} q
\]

This model kinematic is common to all kinds of vehicles which are not omnidirectional (for instance, an automobile, a bicycle, a vehicle with two parallel independent power wheels - power wheeled steering system and a tricycle). The linear velocity and rotational velocity of this kind of vehicle are controlled by their accelerator and steering wheel, respectively.

In this control system two postures are used, the reference posture \( p_r = (x_r, y_r, \theta_r)^T \) and the current posture \( p_c = (x_c, y_c, \theta_c)^T \). The error equation of mobile robot posture can be described as in [10] by (2):

\[
P_e = \begin{bmatrix}
x_c - x_r \\
y_c - y_r \\
\theta_c - \theta_r
\end{bmatrix}
\]

The velocity error of the mobile robot can be obtained as:

\[
\dot{P}_e = \begin{bmatrix}
\dot{x}_e \\
\dot{y}_e \\
\dot{\theta}_e
\end{bmatrix} = \begin{bmatrix}
y_e \omega - v + v_r \cos\theta_e \\
-x_e \omega + v_r \sin\theta_e \\
0 & 0 & 1
\end{bmatrix}
\]

The trajectory tracking problem for the kinematic model of the mobile robot can be described as finding a bounded control input \( q = (v, \omega)^T \) so that given any initial errors to the system (2).

The tracking error \( p_e = (x_e, y_e, \theta_e)^T \) can be bounded and converge asymptotically to zero when \( t \to \infty \), according to the input vector \( q = (v, \omega)^T \).

III. FAST TERMINAL SLIDING MODE CONTROL

A. Finite time tracking control law

In order to design a finite time control law, a first related lemma of first-order linear system is considered [11].

\[
\dot{x} = u
\]

The control \( u \) is written as:

\[
u = -\beta \dot{x}_e^p
\]

with \( \beta > 0 \) and \( p, q (p > q) \) are positive parameters.

The equations (4) and (5) give the above result

\[
\dot{x} = -\beta x_e^p
\]

The dividing integral of the equation (8) in order to calculate the reaching time \( t_r \), the following result can be written.

\[
\int_0^{t_r} dt = \int_0^1 \frac{1}{\beta} x^{-q/p} dx
\]

From any initial state \( x(0) \neq 0 \), the system will arrive at the formula (5) in a limited time \( t_r \) to the reach equilibrium \( x = 0 \) with:

\[
t_r = \frac{p}{\beta(q-p)} |x(0)|^{(p-q)/p}
\]

The goal of the non-linear portion \( \beta x^q/p \) in equation (10) is for improving the balance and convergence speed when the the state is far away from equilibrium. The reaching time permit to the system a fast convergence to the origin [12, 13].

According to the equation (4) of the first-order finite-time the control law \( \omega \) of the system (1) is:

\[
\omega = \omega_r + \beta \theta_e^q/p
\]

By using the equation (4), the above result is obtained

\[
\dot{\theta}_e = -\beta \theta_e^{q/p}
\]

Therefore, the system (2) reaches \( \theta_e = 0 \) in a finite time \( t_f \)

\[
t_f = \frac{p}{\beta(q-p)} |x(0)|^{(p-q)/p}
\]

The system (3) in the finite time \( t_f \) reaches the configuration \( \omega - \omega_l = 0 \)

B. Design of sliding mode tracking control law

At the time \( t_f > t \), the state \( \theta_e \) becomes zero and the equality \( \omega = \omega_l \) is obtained, then just considering another two states of the control design \( x_e \) and \( y_e \). So the system of equation (2) become as:

\[
x_e = \omega_r y_e - v + v_r
\]

\[
y_e = -\omega_l x_e
\]

From the equations (14) and (15), the control law \( v \) is designed by using the traditional sliding mode control technique according to the following steps:
Design switching function

Switching function can be designed as follows:

\[ s = x_e - y_e \]  \hspace{1cm} (16)

By designing sliding mode control law which makes the sliding surface \( S \) converge to zero, it is interesting to consider the convergence of the state \( x_e \) to the state \( y_e \).

In order to achieve \( x_e \) converge to 0 and \( y_e \) converge to 0, it must use the following proof:

Proof:

When \( s = x_e - y_e \), the above Lyapunov function is considered:

\[ V_y = \frac{1}{2} y_e^2 \]  \hspace{1cm} (17)

From equation (17) the derivative is given by:

\[ \dot{V}_y = y_e \dot{y}_e = y_e (-\omega_r x_e) = -\omega_r x_e y_e = -\omega_r y_e^2 \leq 0 \]

This is only when \( x_e = y_e = 0 \) and when the equality is established.

If \( x_e \) and \( y_e \) are equal, then the system state \( x_e \) converges to 0 thus the system state \( y_e \) also converges to zero, which implies that the zero solution \( x_e = 0 \) of equation (15) is asymptotically stable.

Select the reaching law sliding

The reaching law which is defined by (18) is considered.

\[ \dot{s} = K \text{sgn}(s) \]  \hspace{1cm} (18)

In order to reduce the chattering, a continuous function replaces the sign function:

\[ \dot{s} = -K \frac{s}{|s|+\delta} \]  \hspace{1cm} (19)

Sliding mode control law design

The equations (14) and (16) show that:

\[ \dot{s} = -K \frac{s}{|s|+\delta} = \dot{x}_e - \dot{y}_e = \omega_r y_e - v + \omega_r x_e \]  \hspace{1cm} (20)

After finishing, the control law \( v \) is given as follows:

\[ v = v_e + \omega_r y_e + \omega_r x_e + K \frac{s}{|s|+\delta} \]  \hspace{1cm} (21)

Sliding mode tracking control is designed in the direction of the angle error in the case where this last is equal to 0, so that is only when \( \theta_e \) converge to zero and \( \omega = \omega_r \). The sliding mode tracking control law began to play a role, then \( x_e \) converge to 0 and \( y_e \) converge to 0.

For the control law (11) and (21), the parameters \( \beta, p, q, \delta, k \) take values greater than 0, and can ensure that the system is in effective tracking.

IV. SIMULATION RESULTS AND ANALYSIS

To demonstrate the effectiveness of this method, the Matlab environment, considering circular and sinusoidal trajectories, is considered. The desired linear and angular velocities are selected as:

\[ v_r = 2 \text{ m/s} , \omega_r = 1 \text{ rad/s} \]

The reference Posture \( p_r = (x_r, y_r, \theta_r)^T \) is given as follows:

\[
\begin{align*}
x_r &= r \cos(\omega_r t) = \cos(t) \\
y_r &= r \sin(\omega_r t) = \sin(t) \\
\theta_r &= \omega_r t = t
\end{align*}
\]

The parameters are selected as:

\[ \delta = 0.8, k = 2, \beta = 6, p = 10 \text{ and } q = 8 \]

The initial position and orientation tracking error are:

\[ (2m, 1m, \frac{\pi}{6} \text{ rad}) \]

By using the control law (11) and (21), the simulation results are shown in Fig. 2 for the circular trajectory tracking.

Fig. 2. Position tracking of a circular trajectory

The posture errors and the linear and angular control speeds of the robot are presented in Fig. 3 and Fig. 4.
Another simulation with different initial error for the position and the orientation tracking \((3m, 0.5m, \frac{\pi}{3} \text{ rad})\) is realized for circular trajectory and the result is shown in Fig. 5.

For this case, the posture errors and the linear and angular control speeds of the robot are illustrated in Fig. 6 and Fig. 7.

Now, a sinusoidal trajectory is considered with an initial position and orientation tracking errors taken as: \((2m, 1m, \frac{\pi}{6} \text{ rad})\).

By using the control law (11) and (21), the simulation results for a sinusoidal trajectory, concerning the trajectory tracking, are shown in Fig. 8. The posture errors and the linear and angular control speeds of the robot are illustrated in Fig. 9 and Fig. 10.
A Terminal sliding mode control adds nonlinear functions into the design of the sliding surfaces, thus, a terminal sliding surface is constructed and the tracking errors on the sliding surface converge to zero in a finite time. The main feature of the approach is in the design of sliding surfaces such that the error posture of the robot converges to the origin in finite time after reaching the sliding surface. The efficacy of this technique has been successfully demonstrated for mobile robots tracking circular and sinusoidal trajectory. Finally, the theoretical results by achieving a faster convergence rate are validated with simulations works.

REFERENCES


V. CONCLUSION

In this paper, a fast terminal sliding mode control technique for finite-time tracking of mobile robot is presented.